Continuous Hulls of Modules

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Abstract

In a 1982 paper [1], Müller and Rizvi introduced the concept of continuous hulls for modules. A continuous module is one where each submodule is essential in a direct summand (dubbed C1 property) and every submodule isomorphic to a direct summand is itself a direct summand (C2 property). A hull would be an over-module, subject to certain "minimal" property; for instance, in the case of an injective hull $E(M)$ of a module M, by definition there is no other injective module between M and $E(M)$. In the 1982 paper, three definitions, or types, of continuous hulls were proposed and studied; for various cyclic modules over commutative rings such hulls were constructed; intriguingly, it was also shown that such hulls don't always exist.

In this talk, I will present new developments in this direction. Results are extended to finitely generated projective modules over abelian rings, and some explicit constructions are given. Classes of modules which have continuous hulls over a commutative ring are studied.

Furthermore, certain classes of rings have the property that every continuous module is quasi-injective; for such classes of rings, the continuous hull of any module is exactly its quasi-injective hull which always exists. I will present results concerning commutative rings with this property, and also in contrast, examples of modules for which the continuous hull is distinct from its (quasi-)injective hull as well as its quasi-continuous hull.

References

[1] B. J. Müller, S. T. Rizvi: *On the existence of continous hulls.* Comm. Alg. 10 (17), pp. 1819–1838, 1982.